ABSTRACT

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The well-known Volume Conjecture of Kashaev, Murakami, and Murakami posits that the growth rate of the colored Jones polynomial of a link encodes the simplicial volume of its complement. Analogous conjectures have been made for other quantum invariants of links and three-manifolds, and a growing body of evidence suggests a relationship between these invariants and three-dimensional geometry.

We define a quantum invariant of links in the thickened torus, which we call the toroidal colored Jones polynomial. Like the usual colored Jones polynomial, our invariant can be defined skein-theoretically, via the skein module of the torus, or using the theory of operator invariants. Most interestingly, our toroidal colored Jones polynomial exhibits volume conjecture behavior. We prove volume convergence for the two-by-two square weave, and show that a suitably defined volume conjecture for our invariant implies the original Volume Conjecture.